Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name: Ring Theory

Subject Code: 4SC06RIT1 **Branch: B.Sc.(Mathematics)**

Semester: 6 **Date : 30/04/2019** **Time : 10:30 To 01:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Define: Boolean Ring (01)

b) What do you mean by right cosets? (01)

c) Give an example of division ring . (01)

d) True/false: Every integral domain is field. (01)

e) Define : Subring. (01)

f) State Division Algorithm for polynomials. (01)

g) Write the unit element of $Z_5 - \{0\}$. (01)

h) What is difference between unit element and unity? (01)

i) Is $Z_6 - \{0\}$ becomes field ? justify your answer . (02)

j) State unique factorization of polynomial . (02)

k) What is g.c.d of $p(x)$ and (x) ? (02)

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) State and properties of ring. **(05)**

b) Let $(M_2(Z);\ +;\cdot)$ be a ring. Check whether the $I = \left\{\begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix} \middle| a, b \in \boldsymbol{Z}\right\}$ is an ideal of $M_2(Z)$ or not. **(05)**

c) Prove that field is an integral domain. **(04)**

Q-3 **Attempt all questions** **(14)**

a) Find the g.c.d of $f(x) = 6x^3 + 5x^2 - 2x + 25$ and $g(x) = 2x^2 - 3x + 5$ $\in R[x]$ and express it in the form $a(x) \cdot f(x) + b(x) \cdot g(x)$. **(05)**

b) If a commutative ring R with unity has no proper ideal, then prove that R is a field. **(05)**

c) A ring R is commutative if $a^2 = a$ for each $a \in R$. **(04)**

Q-4 **Attempt all questions** **(14)**

a) A non empty subset U of a ring R is a subring of R if and only if the following conditions are satisfied. **(05)**

i) $a - b \in U$ and ii) $ab \in U$ for $a, b \in U$

b) A homomorphism defined on the ring $(Z;+;\cdot)$ is either a zero homomorphism or identity mapping. **(05)**

c) Show that intersection of two subring is again subring. **(04)**

Q-5 **Attempt all questions** **(14)**

a) If we define addition and multiplication on power set $P(U)$,U being the universal set as follows, for $A, B \in P(U)$,
$A + B = A\Delta B = (A \cup B) - (A \cap B)$ and $AB = A \cap B$ then show that $(P(U), +,\cdot)$ is a ring. **(05)**

b) Suppose $(R;+;\cdot)$ is a ring with unity. Define addition $\oplus$ and multiplication $\odot$ in R as follows $a\oplus b = a + b + 1$ and $a\odot b = a + b + ab$ for$a, b \in R$, then show that $(R;\ \oplus;\odot)$ is a ring. **(05)**

c) The characteristic of an integral domain is either a prime number or zero. **(04)**

Q-6 **Attempt all questions** **(14)**

a) Obtain all principal ideals in the ring $(Z_{12};\ +_{12};\cdot_{12})$. **(05)**

b) Let $I = 8Z$ in the ring $R = (2Z;\ +\ ;\cdot)$ prepare addition and multiplication table for the quotient ring $R\backslash I$. **(05)**

c) The characteristic of a ring R with unity is n if and only if n is the smallest positive integer with $n \cdot 1 = 0$. **(04)**

Q-7 **Attempt all questions** **(14)**

a) Let $(R;+;\cdot)$ be a ring with unity then prove that the mapping $\phi: (Z;+;\cdot) \rightarrow (R;+;\cdot)$,where $\phi(n) = n \cdot 1, n \in Z$ is homomorphism with **(05)**

i) $K_{\phi} =< m >$, if the characteristic of R is m.

ii) $K_{\phi} = \{0\}$, if the characteristic of R is zero.

b) Let I_1, I_2 be an ideal of ring R then show that $I_1 \cup I_2$ is an ideal of R if and only if either $I_1 \subset I_2$ or $I_2 \subset I_1$. **(05)**

c) Show that $(Z;+;\cdot)$ is a principal ideal ring. **(04)**

Q-8 **b)** **Attempt all questions** **(14)**

c) For a non zero polynomials$f, g \in D[x], [f \cdot g] = [f] + [g]$. **(05)**

b) For associate polynomials $f(x), g(x) \in F[x]$, then show that $f(x) = cg(x)$ for some $c \neq 0$ and$c \in F$. **(05)**

c) Prove that Division algorithm is not true in $Z[x]$. **(04)**